GCSE Maths - Algebra

Expanding Brackets

Notes

WORKSHEET

Expanding Brackets

Sometimes it is necessary to **multiply out brackets** of an expression. This process is called **expanding brackets.** This allows you to simplify the expression by **collecting like terms.**

First, we will look at multiplying out **single brackets**. There are a few key things that have to be remembered when multiplying out single brackets.

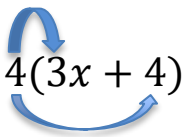
Expanding Single Brackets

- The terms **outside** the bracket will be separately multiplied with each of the **individual terms** that are **inside** the bracket.
- If there are **two or more letters** that are multiplied together, they are **written together** next to each other and in **alphabetical order**
  - $a \times b = ab$
  - $x \times z \times y = xyz$

  You can think of there being an invisible multiplication sign between the letters.
- When **two of the same letters** are multiplied, the **power** of the letter **increases:**
  - $a \times a = a^2$
  - $(ab)^2 = a \times a \times b \times b = a^2b^2$

Example: Expand the expression $4(3x + 4)$

1. **Draw arrows** from the term **outside the bracket to each of the terms inside the bracket**, to show which multiplications need to be carried out.

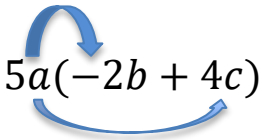


2. Compute **multiplications** and **sum** them together.

$$4(3x + 4) = [4 \times 3x] + [4 \times 4] = \mathbf{12x + 16}$$

Example: Expand the expression $5a(-2b + 4c)$

1. **Draw arrows** from the term **outside the bracket to each of the terms inside the bracket**, to show which multiplications need to be carried out.

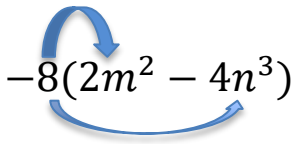


2. Compute **multiplications** and **sum** them together.

$$5a(-2b + 4c) = [5a \times -2b] + [5a \times 4c] = \mathbf{-10ab + 20ac}$$

Example: Expand the expression $-8(2m^2 - 4n^3)$

1. **Draw arrows** from the term **outside the bracket to each of the terms inside the bracket**, to show which multiplications need to be carried out.



2. Compute **multiplications** and **sum** them together.

$$-8(2m^2 - 4n^3) = [-8 \times 2m^2] + [-8 \times -4n^3] = \mathbf{-16m^2 + 32n^3}$$

Expanding Double Brackets

Multiplying two brackets together is slightly different to expanding single brackets, as now we need to multiply **everything** that is in the **first bracket** by **everything** that is in the **second bracket.**

FOIL method

One way of ensuring that you multiply everything in the first bracket with everything in the second bracket is by using the **FOIL method.**

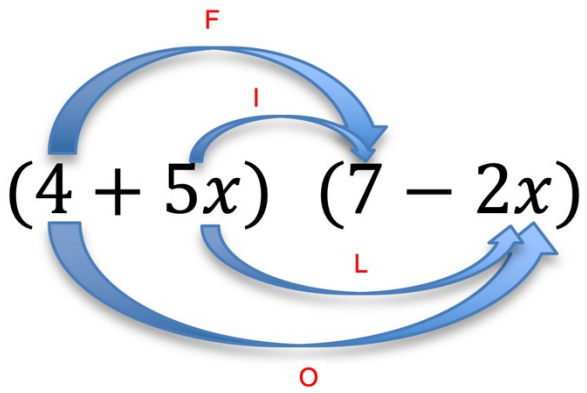
First: Multiply the **first terms in each bracket** with each other.

Outside: Multiply the outside terms. This means you multiply the **first term in bracket 1** with the **second term in bracket 2**.

Inside: Multiply the inside terms. This means you multiply the **second term in bracket 1** with the **first term in bracket 2.**

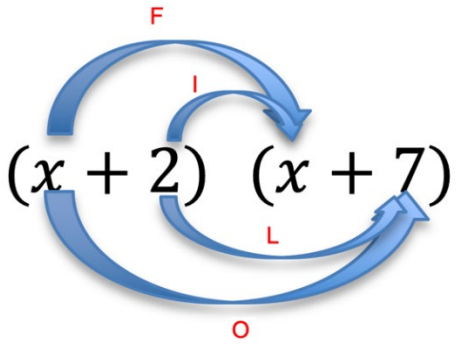
Last: Multiply the last terms. This means you multiply the **second terms in each bracket with each other.**

Example of what FOIL may look like:



Example: Expand the expression $(x + 2)(x + 7)$

1. Expand the expression using the **FOIL method**. You can **draw arrows** to ensure that you have multiplied all the terms in bracket 1 with all of the terms in bracket 2.



2. Write **each multiplication** and **sum them together**.

F: $x \times x = x^2$
O: $x \times +7 = +7x$
I: $+2 \times x = +2x$
L: $+2 \times +7 = +14$

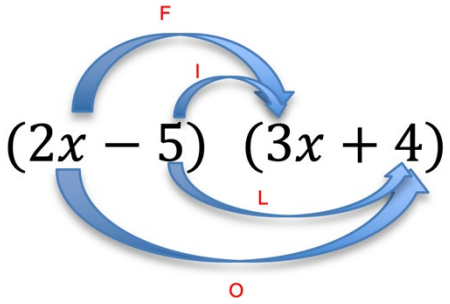
$$(x + 2)(x + 7) = x^2 + 7x + 2x + 14$$

3. **Collect** any **like terms**.

$$(x + 2)(x + 7) = \boldsymbol{x^2 + 9x + 14}$$

Example: Expand the expression $(2x - 5)(3x + 4)$

1. Expand the expression using the **FOIL method**. You can **draw arrows** to ensure that you have **multiplied all the terms in bracket 1 with all of the terms in bracket 2.**



2. Write **each multiplication** and **sum them together**.

F: $2x \times 3x = 6x^2$
O: $2x \times +4 = 8x$
I: $-5 \times 3x = -15x$
L: $-5 \times +4 = -20$

$$(2x - 5)(3x + 4) = \boldsymbol{6x^2 + 8x - 15x - 20}$$

3. **Collect** any **like terms**.

$$(2x - 5)(3x + 4) = \boldsymbol{6x^2 - 7x - 20}$$

Square Brackets and the Difference of Two Squares

Square Brackets

Square brackets refer to an expression in the general form of $(\boldsymbol{ax+b})^2$. To solve this, write the initial square brackets as **two separate brackets**:

$$(\boldsymbol{ax+b})^2 = (\boldsymbol{ax+b})(\boldsymbol{ax+b})$$

Now you can rewrite the expression by **expanding the two brackets** using the **FOIL method**

Difference of Two Squares

When expanding an equation in the general form of $(\boldsymbol{a+b})(\boldsymbol{a-b})$, we can use the **FOIL method**. The answer will lead to an expression in the general form

$$(\boldsymbol{a+b})(\boldsymbol{a-b}) = \boldsymbol{a^2 - b^2}$$

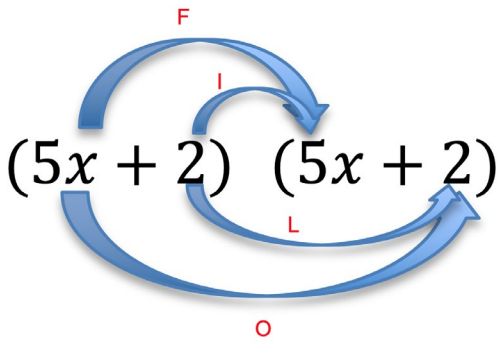
This is known as the difference of two squares where we have an expression with one squared term subtracted from another squared term.

Example: Expand the expression $(5x+2)^2$

1. Write the expression as the product of two brackets.

$$(5x+2)^2 = (5x+2)(5x+2)$$

2. **Expand** the expression using the **FOIL method**. **You can draw arrows** to ensure you have multiplied all the terms in bracket 1 with all of the terms in bracket 2.



3. Write **each multiplication** term and **add them up**.

F: $5x \times 5x = 25x^2$
O: $5x \times +2 = 10x$
I: $+2 \times 5x = 10x$
L: $+2 \times +2 = 4$

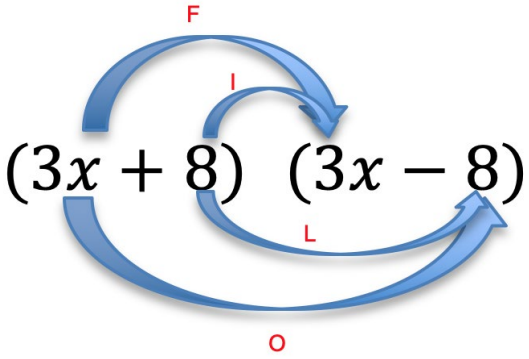
$$(5x+2)^2 = \boldsymbol{25x^2 + 10x + 10x + 4}$$

4. **Collect** any **like terms**.

$$(5x+2)^2 = 25x^2 + 10x + 10x + 4 = \boldsymbol{25x^2 + 20x + 4}$$

Example: Expand the expression $(3x + 8)(3x - 8)$

1. **Expand** the expression using the **FOIL method**. **You can draw arrows** to ensure you have multiplied all the terms in bracket 1 with all of the terms in bracket 2.



2. Write **each multiplication** term and **add them up**.

F: $3x \times 3x = 9x^2$
O: $3x \times -8 = -24x$
I: $+8 \times 3x = +24x$
L: $+8 \times -8 = -64$

$$(3x + 8)(3x - 8) = \mathbf{9x^2 - 24x + 24x - 64}$$

3. **Collect** any **like terms**.

$$(3x + 8)(3x - 8) = 9x^2 - 24x + 24x - 64 = \mathbf{9x^2 - 64}$$

Note: The expression in the question follows the general form of $(a + b)(a - b)$ where in this example $a = 3x$ and $b = 8$. The formula $(a + b)(a - b) = a^2 - b^2$ means that we know the answer must be $(3x)^2 - 8^2 = 9x^2 - 64$.

Expanding Three or More Brackets (Higher only)

It is possible that you could be asked to expand three or more brackets. The process is much the same as with two brackets.

- If there are three or more brackets, the simplest way to expand is to first expand two of the brackets using the **FOIL** method.
- Once you have expanded the first two brackets, multiply this answer with another one of the brackets found in the original question. This time the FOIL method will not always work as one of the brackets may contain more than 2 terms. The easiest way is to systematically take each term from the first bracket and multiply it with each of the terms in the second bracket.
- **Repeat** for how many times this is needed. If it is a triple bracket it will not need to be repeated again as you will have now expanded all the brackets.

Example: Expand the expression $(3x+2)(2x+1)(4x+2)$

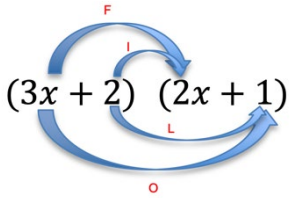
1. Pick two of the brackets.

   For simplicity's sake we will pick the first two brackets. We will group these two together but keep the remaining third bracket there so we don't forget about it:

   $$(3x+2)(2x+1)(4x+2) = [(3x+2)(2x+1)](4x+2)$$

2. Expand the first two brackets using the FOIL method and add up the terms.

   F: $3x \times 2x = 6x^2$
   O: $3x \times +1 = +3x$
   I: $+2 \times 2x = +4x$
   L: $+2 \times +1 = +2$

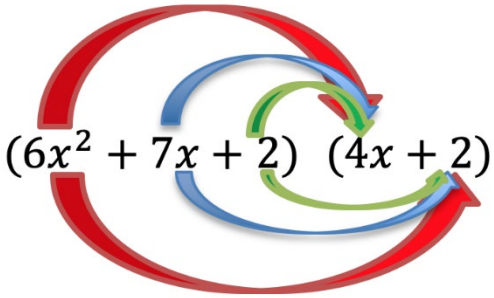
   

   $(3x+2)(2x+1) = 6x^2 + 3x + 4x + 2 = \mathbf{6x^2 + 7x + 2}$

3. Multiply the quadratic $6x^2 + 7x + 2$ with the remaining bracket $(4x+2)$.

   $$(3x+2)(2x+1)(4x+2) = (6x^2 + 7x + 2)(4x+2)$$

   We **CANNOT** use the FOIL method as there are three terms in the first bracket. However, we can expand it by multiplying each term in the first bracket with each term in the second bracket.

   It is colour-coded to make it clear that every term in the first bracket is multiplied with every term in the second bracket. Now write **each multiplication** and **add them up**.

   RED: $6x^2 \times 4x = 24x^3$
   RED: $6x^2 \times +2 = +12x^2$
   BLUE: $+7x \times 4x = +28x^2$
   BLUE: $+7x \times +2 = +14x$
   GREEN: $+2 \times 4x = +8x$
   GREEN: $+2 \times +2 = +4$

   $$(6x^2 + 7x + 2)(4x+2) = \mathbf{24x^3 + 12x^2 + 28x^2 + 14x + 8x + 4}$$

4. Collect any like terms.

   $$24x^3 + 12x^2 + 28x^2 + 14x + 8x + 4 = \mathbf{24x^3 + 40x^2 + 22x + 4}$$

Expanding Brackets – Practice Questions

1. Expand the following expressions:

   a) $4a(2b + 2)$
   b) $6p(7q - 8)$
   c) $-3a(-10d + 6a)$
   d) $8x(-2x + 3y)$

2. Expand the following expressions:

   a) $(x + 1)(x + 8)$
   b) $(a + 2)(b - 3)$
   c) $(2p + 5)(p + 1)$
   d) $(x - 4)(3y + 2)$

3. Expand the following expressions:

   a) $(5a + 4)^2$
   b) $(-2 + 6p)^2$

4. Expand the following expressions:

   a) $(a + 9)(a - 9)$
   b) $5(7 - 8x)(7 + 8x)$

5. Expand the following expressions:

   a) $(x + 1)(x + 3)(x - 4)$
   b) $(5y + 2)(y - 9)(2y + 8)$

Worked solutions for the practice questions can be found amongst the worked solutions for the corresponding worksheet file.